# Formation of solitary waves and shocklets in a two-temperature electron κ distributed plasma


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## Abstract

- Large-amplitude electron acoustic (EA) waves and shocklets are investigated in a two-temperature electron plasma.
- Dynamical cold electrons are described by the fully nonlinear continuity and momentum equations.
- Superthermal (hot) inertialess electrons are assumed to follow κ distribution in a background of static positive ions.
- The fluid equations along with quasineutrality equation are solved to obtain a set of two characteristic wave equations that admit both analytical and numerical solutions.
- Variation due to hot electron superthermality and hot to cold electron density ratio strongly affects the nonlinear structures involving the negative potential, cold electron velocity and density profiles.
- For τ=0 symmetric solitary pulses are formed, which are developed into shocklets with the course of time.


## Electron-acoustic Waves

- EA waves occur in a two-temperature electron plasma containing hot and cold electron components neutralized by singly ionized positive ions.
- The wave oscillation frequency is higher than the ion plasma frequency.
- Phase velocity of the wave intermediates the cold and hot electron thermal speeds.
- On a cold electron timescale, positive ions assumed to be immobile only appearing in the equilibrium charge-neutrality condition, while inertialess hot electron pressure provides a restoring force to maintain the EA wave.
- In 1977, Watanabe and Taniuti established a theoretical study for EA waves to confirm its propagation in a collisionless unmagnetized two temperature electron plasma.
- The Fast Auroral SnapshoT (FAST) observations in the auroral regions (altitude < 4000 km), geotail, and the polar observations at higher altitude (between ~2$R_E$ and 8$R_E$, $R_E$ being earth's radius) auroral region confirm the existence of EAWs in several parts of magnetosphere
- EA waves are also observed in laser produced plasma

## Governing Equations

- Non Maxwellian unmagnetized collisionless plasma
  - dynamical cold electrons
  - interialess superthermal hot electrons
  - uniformly distributed immobile positive ions
- Governing nonlinear 1D fluid equations for EA waves

$$\frac{\partial n_c}{\partial t}+\frac{\partial}{\partial x}(n_c u_c)=0 \quad \text{Continuity equation}$$

$$\left(\frac{\partial}{\partial t}+u_c\frac{\partial}{\partial x}\right)u_c=\frac{e}{m_e}\frac{\partial\phi}{\partial x} \quad \text{Equation of motion}$$

$$\frac{\partial^2\phi}{\partial x^2}=-4\pi e(n_i-n_c-n_h) \quad \text{Poisson equation}$$

$$n_h(\phi)=n_{ho}\left\{1-\left(\kappa-\frac{3}{2}\right)^{-1}\frac{e\phi}{k_BT_h}\right\}^{-\kappa+\frac{1}{2}} \quad \text{superthermal (hot) electron density distribution}$$

- The above set of equations can be normalized as.

$$\left(\frac{\partial}{\partial\tau}+U_c\frac{\partial}{\partial X}\right)N_c+N_c\frac{\partial U_c}{\partial X}=0 \quad (1) \qquad \left(\frac{\partial}{\partial\tau}+U_c\frac{\partial}{\partial X}\right)U_c-\frac{\partial\Phi}{\partial X}=0 \quad (2)$$

$$N_c=1+\beta-\beta[1-\Phi/(\kappa-3/2)]^{-\kappa+1/2} \quad (3)$$

- Scaled parameters used $\tau=t\omega_{pc}$ $X=x/\lambda_0$ $N_c=n_c/n_{c0}$ $U_c=u_c/v_{Th}$ $\Phi=e\phi/k_BT_h$

where $\beta=(n_{i0}/n_{c0})-1$ $\beta=n_{h0}/n_{co}$ $v_{Th}=(k_BT_h/m_e)^{1/2}$ $\lambda_0=(k_BT_h/4\pi e^2 n_{c0})^{1/2}$ $\omega_{pc}=(4\pi n_{c0}e^2/m_e)^{1/2}$

## Nonlinear Equations for large amplitude EA waves

By Substituting Eq. (3) into Eq. (1) and performing differentiations with respect to X and τ results the following set of nonlinear equations

$$\left(\frac{\partial}{\partial\tau}+U_c\frac{\partial}{\partial X}\right)U_c-\frac{\partial\Phi}{\partial X}=0 \quad (4) \qquad \frac{\partial\Phi}{\partial\tau}+U_c\frac{\partial\Phi}{\partial X}-\chi(\Phi)\frac{\partial U_c}{\partial X}=0 \quad (5)$$

writing nonlinear equations (4) and (5) in matrix form

$$\partial_\tau\begin{bmatrix}\Phi\\U_c\end{bmatrix}+\begin{bmatrix}U_c & -\chi(\Phi)\\-1 & U_c\end{bmatrix}\partial_X\begin{bmatrix}\Phi\\U_c\end{bmatrix}=0 \quad (6)$$

where $\chi(\Phi)=\dfrac{1+\beta-\beta[1-c_\kappa\Phi/(\kappa-1/2)]^{-\kappa+1/2}}{c_\kappa\beta[1-c_\kappa\Phi/(\kappa-1/2)]^{-\kappa-1/2}}$ $\quad c_\kappa=(2\kappa-1)/(2\kappa-3)$

## Diagonalization of Matrix

Solution of nonlinear equation is found by following the **Diagonalization of Matrix** formalism. In order to diagonalize a matrix as a first step compute $\det(A-\lambda I)=0$ to obtain the eigen values as $\lambda_\pm=U_c\pm\sqrt{\chi(\Phi)}$

where $A=\begin{bmatrix}U_c & -\chi(\Phi)\\-1 & U_c\end{bmatrix}$ and $I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

Determine diagonalizing matrix C such that $D=C^{-1}AC$ where D is a diagonal matrix consisting of eigen values on the main diagonal and zeros everywhere and C is a matrix consisting of column eigen vectors.

$$D=C^{-1}AC=\begin{bmatrix}\lambda_+ & 0\\0 & \lambda_-\end{bmatrix} \qquad C=\begin{bmatrix}1 & 1\\-\frac{1}{\sqrt{\chi(\Phi)}} & \frac{1}{\sqrt{\chi(\Phi)}}\end{bmatrix}$$

Multiply Eq.(6) by $C^{-1}$ in the left-hand side to obtain $\dfrac{\partial\Psi_\pm}{\partial\tau}+\lambda_\pm\dfrac{\partial\Psi_\pm}{\partial X}=0$ where $\Psi_\pm=U_c\mp F(\Phi)$ with $F(\Phi)=\int_0^\Phi\left\{\frac{1}{\chi(\Phi)}\right\}^{\frac{1}{2}}d\Phi$

Generally, $\Psi_+$($\Psi_-$) represents a wave propagating along the positive (negative) x direction. To find wave solution, set $\Psi_+$ or $\Psi_-$ equal to zero taking $\Psi_-=0$ gives $U_c=-F(\Phi)$ which further leads to $\Psi_+=2U_c$

Hence, wave equation can be expressed in terms of $U_c$ and Φ, as

$$\frac{\partial U_c}{\partial\tau}+\lambda_+(\Phi)\frac{\partial U_c}{\partial X}=0 \qquad \frac{\partial\Phi}{\partial\tau}+\lambda_+(\Phi)\frac{\partial\Phi}{\partial X}=0 \quad (7)$$

where eigen value (effective phase speed) $\lambda_+(\Phi)=-F(\Phi)+\sqrt{\chi(\Phi)}$

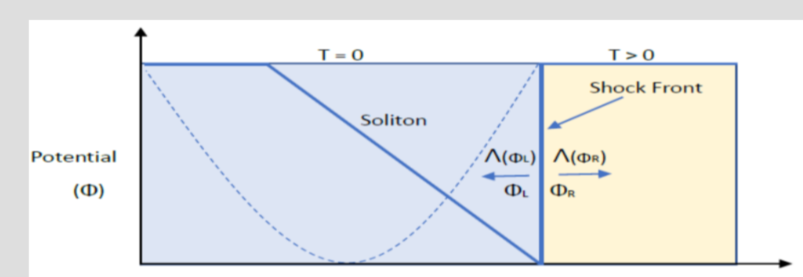
General solution for Eq.(7) is given by $\Phi=\Phi_0[X-\lambda_+(\Phi)\tau]$

- Shocklets propagate with speed under Rankine- Hugoniot condition

$$v(shock)=[\Gamma_L(\Phi_L)-\Gamma_R(\Phi_R)/(\Phi_L-\Phi_R)]$$

$\lambda_+(\Phi)=c_{0\kappa}+B_\kappa\Phi$ $\qquad v(shock)=[c_{0\kappa}+(B_\kappa\Phi_L)/2]$

where $B_\kappa=-\sqrt{\beta c_\kappa}(3/2+d_\kappa/2\beta)$ $\quad d_\kappa=(2\kappa+1)/(2\kappa-1)$ $\quad c_{0\kappa}=1/\sqrt{\beta c_\kappa}$




## Results and Discussion

- We have investigated the time evolution profiles of large amplitude non-stationary EA waves and numerically solved Eqn.(7) by setting an initial condition for the localized electrostatic potential as $\Phi=\Phi_m$sech[X/d] with its amplitude $\Phi_m$=-0.15 and pulse width d=5.
- We have chosen some typical numerical values of non-Maxwellian laboratory plasma e.g., $n_{h0}=7\times10^7$cm$^{-3}$, $n_{c0}=2\times10^7$cm$^{-3}$, $T_c$=0.7 eV, $T_h$=2.1eV, $\omega_{pc}=2.52\times10^8$ rad/s, $v_{Tc}=3.50\times10^7$cm/s, $v_{Th}=6.07\times10^7$cm/s and $\lambda_0$=0.24cm
- It is observed that the effective phase speed become large at a small value of κ.
- It is shown that at τ=0 potential pulses overlap but this symmetry breaks for τ >0.
- Solitary pulses develops into shocks for τ >0 with increase self steepness and wave amplitude.
- The variation of the hot electron superthermality effects reduces the solitary and shock wave amplitudes and pulse widths with fixed hot to cold electron density ratio.
- Increase in hot to cold electron density ratio with fixed value of κ results an increase in the magnitude solitary and oscillatory shock waves.

## References


1. B. Eliasson and P. K. Shukla, Phys. Rev. E 69, 067401 (2004) 2. K. Watanabe and T. Taniuti, J. Phys. Soc. Jpn. 43, 1819 (1977).
3. S. P. Gary and R. L. Tokar, Geophys. Res. Lett. 11, 1180 (1984); Phys. Fluids. 28, 2439 (1985).
4. M.A. Hellberg,R.L.Mace,R.J.Armstrong and G.Karlstad,J. Plasma.Phys.64, 433 (2000).


## Numerical Results of EA-Soliton and Shock

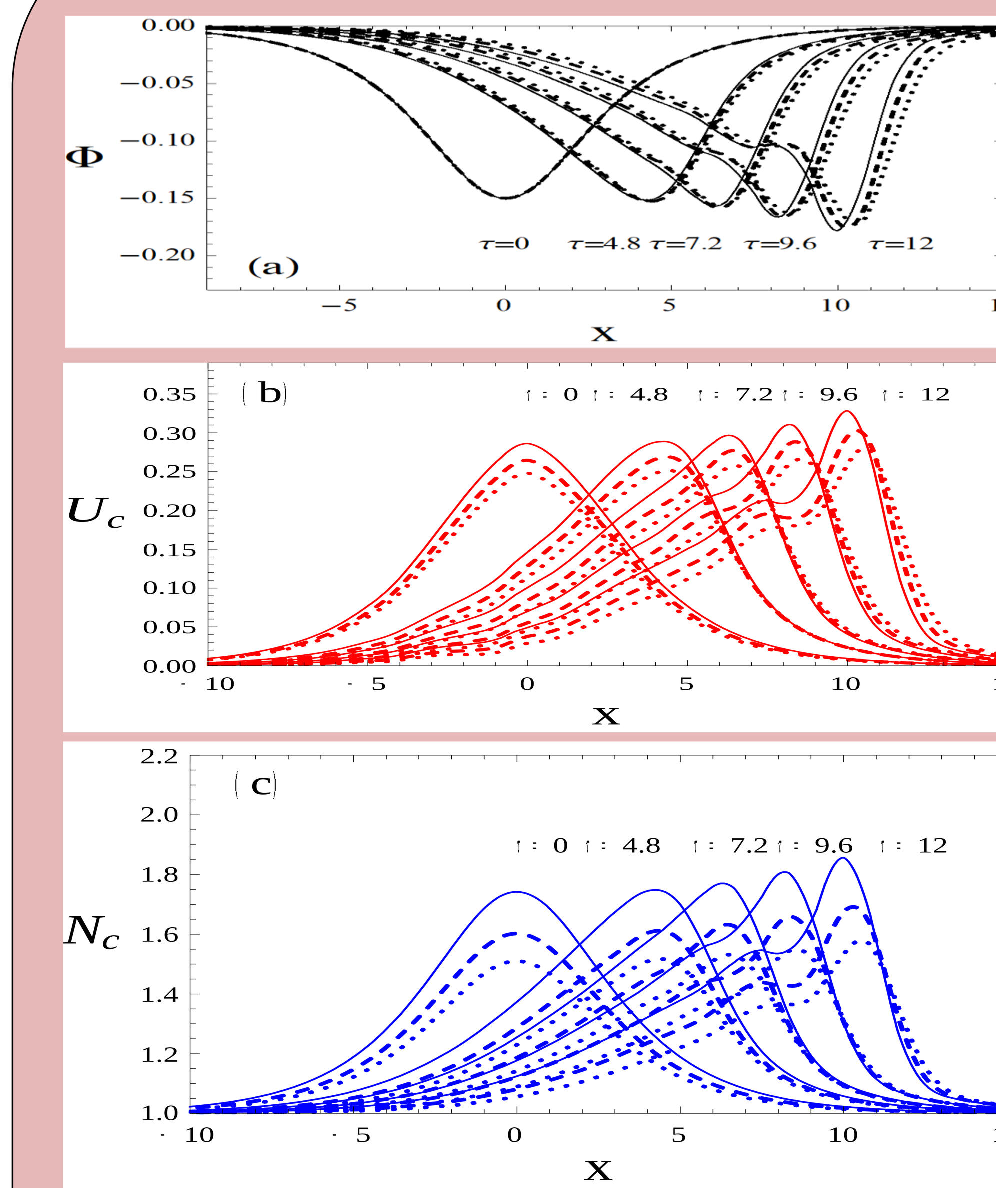



Fig 1: The time evolution of the normalized (a) negative potential Φ (b) cold electron velocity $U_c$ and (c) cold electron density $N_c$ is plotted against the normalized position X for different values of $\kappa=3$ (solid curve), $\kappa=5$ (dashed curve), and $\kappa=20$ (dotted curve) with $\beta=3.5$ and $T_h=2.1eV$.

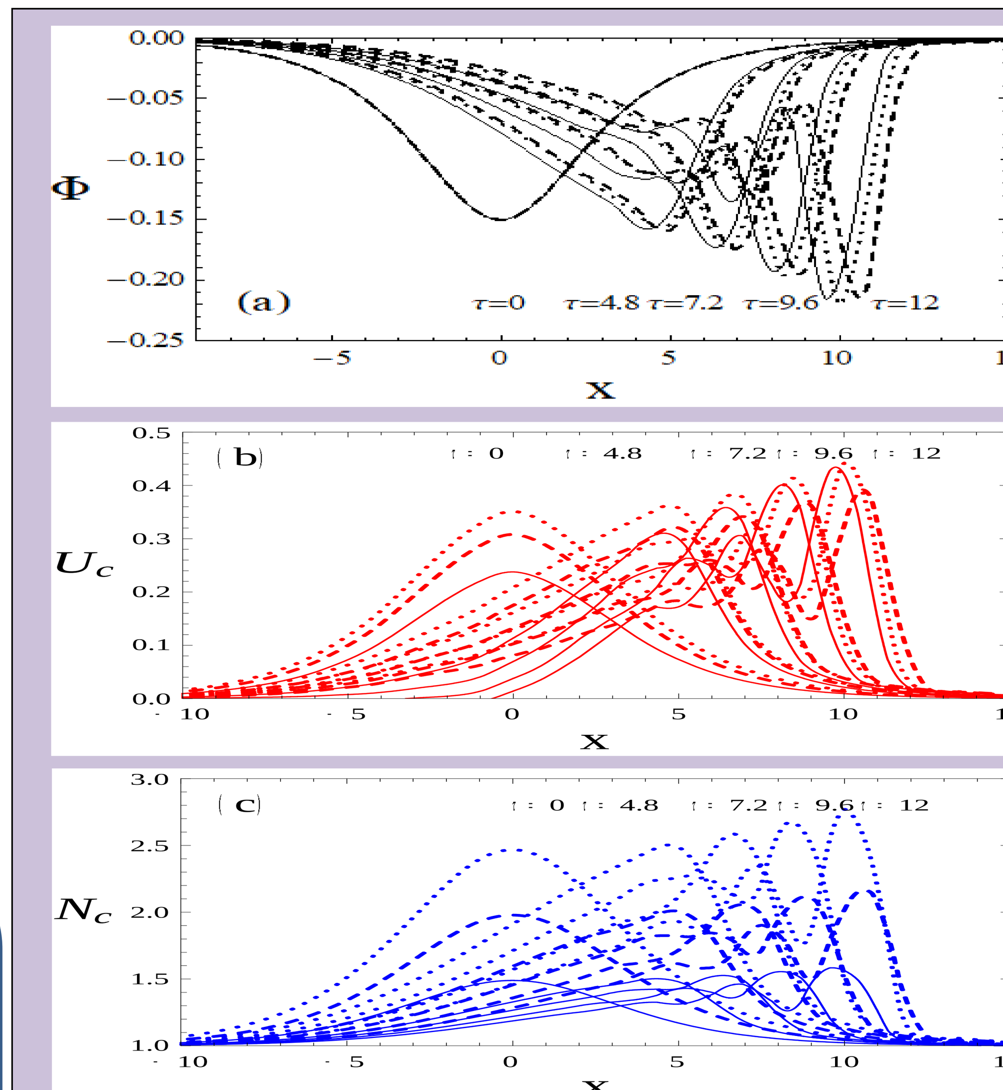



Fig 2: The time evolution of the normalized (a) negative potential Φ (b) cold electron velocity $U_c$ and (c) cold electron density $N_c$ is plotted against the normalized position X for different values of $\beta=1$ (solid curve), $\beta=2$ (dashed curve), and $\beta=3$ (dotted curve) with $\kappa=1.7$ and $T_h=2.1eV$.

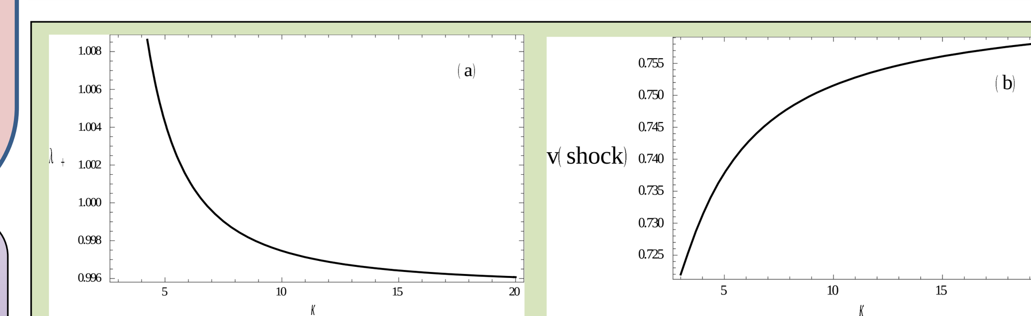



Fig 3: (a) The normalized effective phase speed (b) normalized shock speed v(shock) are plotted against the kappa parameter in the range 3 ≤κ ≤20 with fixed β=3.5 and Φ= $\Phi_L$=-0.15